The UNNS Gauge Protocol (UGP): Connections, Curvature, and Recursive Field Strengths

UNNS Research Notes

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Abstract

Building on the UNNS Vector Protocol (UVP) and Tensor Protocol (UTP), we develop the *UNNS Gauge Protocol* (UGP). This framework treats UNNS operators as connections, defines curvature through operator commutators, and interprets repair/normalization as gauge fixing. We illustrate parallels with Maxwell and Yang–Mills theory, showing how recursion generates field-like structures in UNNS.

Contents

1 Motivation

Gauge theory is the natural language of modern physics, capturing electromagnetism, Yang–Mills fields, and general relativity. To position UNNS as a mathematical substrate for recursive physics, we introduce a gauge protocol where operators act as connections and their commutators define curvature.

2 UNNS Connections

Definition 2.1 (UNNS Connection)**.** *Let $\mathbb{V}$ be the UNNS vector space of nests. A* UNNS connection *is an operator-valued map*

$$\mathcal{A} : \mathbb{V} \to \mathrm{End}(\mathbb{V}),$$

assigning to each vectorized nest $v \in \mathbb{V}$ an operator $\mathcal{A}(v)$ from the Dodecad.

Remark 2.2. *This mirrors how gauge theory assigns Lie algebra elements to tangent vectors.*

3 Curvature as Field Strength

Definition 3.1 (Recursion Curvature)**.** *The curvature of a UNNS connection $\mathcal{A}$ is*

$$\mathcal{F}(v, w) = [\mathcal{A}(v), \mathcal{A}(w)] - \mathcal{A}([v, w]),$$

where $[\cdot, \cdot]$ denotes commutator.

Lemma 3.2. *If operators commute, $\mathcal{F} = 0$, corresponding to a flat connection.*

Remark 3.3. *Nonzero curvature measures recursive instability or interaction, analogous to electromagnetic or Yang–Mills fields.*

4 Gauge Transformations

Definition 4.1 (Gauge Transformation)**.** *A gauge transformation is a map $g : \mathbb{V} \to G$, where G is a group of admissible nest automorphisms, such that*

$$\mathcal{A} \mapsto g\mathcal{A}g^{-1} + g\,dg^{-1}.$$

Remark 4.2. *In UNNS, g corresponds to* repair or normalization*, stabilizing recursion while preserving equivalence.*

5 Theorems

Theorem 5.1 (Gauge Invariance)**.** *Curvature $\mathcal{F}$ is invariant under gauge transformations.*

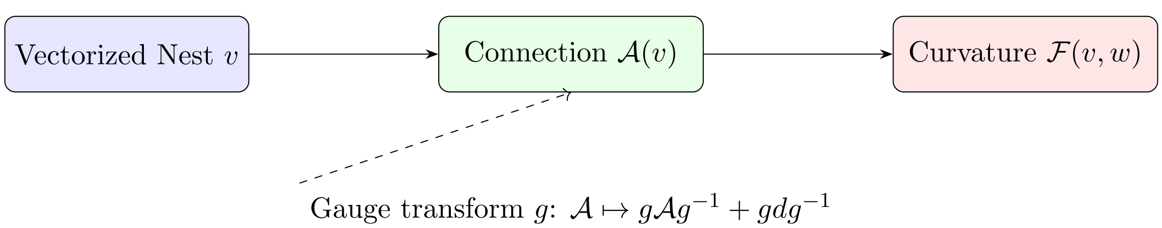
Proof. Follows from the standard gauge theory identity $\mathcal{F} \mapsto g\mathcal{F}g^{-1}$, since commutators transform covariantly. □

Theorem 5.2 (Bianchi Identity in UNNS)**.** *The recursion curvature satisfies*

$$d\mathcal{F} + [\mathcal{A}, \mathcal{F}] = 0,$$

ensuring consistency of recursive propagation.

6 Diagrammatic Overview



7 Applications

7.1 Mathematics

- Defines recursion curvature classes as analogues of Chern classes.
- Opens path to UNNS-based topological field theories.

7.2 Physics

- Maxwell's equations emerge for abelian UNNS curvature.
- Yang–Mills theory arises for non-abelian Dodecad commutators.

7.3 Computation

- Enables error correction as gauge fixing.
- Suggests machine learning models treating normalization as gauge symmetry.

8 Conclusion

The UNNS Gauge Protocol equips recursion with the full machinery of gauge theory: connections, curvature, and gauge invariance. It provides a bridge between recursive mathematics and physical field theories, suggesting new avenues for both rigorous mathematics and speculative physics.